

科目	工程數學	適用系所	積體電路產業研發碩士專班	時間	100分鐘
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※請務必在答案卷作答區內作答。 共 1 頁第 1 頁

1.) 1. If  $f(x) = \frac{\sin x}{x}$ , determine the Laplace transform  $F(s) = L\{f(x)\}$ . (10%)

2. By using the result in 1, to evaluate  $\int_0^{\infty} \frac{\sin x}{x} dx = ?$  (5%)

2.) Consider the **series solution** of the following differential equation at the point  $x = 2$

$$x(x-2)y'' + 2(x+1)y' + 4y = 0$$

1. The solution  $y(x)$  can be expressed as:  $y(x) = \underline{\hspace{2cm}}$ . (Note: Do not give the detail solution) and the region of convergence is  $\underline{\hspace{2cm}}$ . (8%)

2. Determine the indicial equation. (7%)

3.) Consider the following differential equation

$$(x-3)^2 y'' - 3(x-3)y' + 4y = 0$$

1. Solve the differential equation. (5%)

2. Verify your answer in 1, by using the series method at the point  $x = 3$ . (15%)

4.) Find the eigenfunctions expansion of the given function in the eigenfunctions of the following Sturm-Liouville problem:  $f(x) = 1 - x$  for  $0 \leq x \leq L$ , and  $y'' + \lambda y = 0$  with  $y'(0) = 0$  and  $y(L) = 0$  (15%)

5.) Calculate  $\int_0^{\infty} \frac{x^2 \cos 3x}{(x^2 + 1)^2} dx = ?$  (15%)

6.) Find  $u(x,y) = ?$  If  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial x} + 2$ , and  $u(0,y) = 0$ ,  $\frac{\partial u(x,0)}{\partial x} = x^2$  (20%)