

科目	工程數學	適用系所	積體電路產業研發碩士專班	時間	100分鐘
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※請務必在答案卷作答區內作答。 共 1 頁第 1 頁

1.) If $F(s) = \frac{3s+2}{(s+2)(s+5)} = L\{f(t)\}$ (15%)

- Determine $f(0)$ and $f(\infty)$ by using initial and final value theorem. (5%)
- Find $f(t) = L^{-1}\{F(s)\}$ and verify your answer in (1) at $t = 0$ and $t = \infty$ (10%)

2.) Consider the following differential equation (15%)

$$x^2 y'' + xy' - y = 0$$

- Solve the differential equation. (5%)
- Verify your answer in (1) by using power series method at the point $x = 0$. (10%)

3.) Consider the following complex function (20%=5%×4)

$$f(z) = 2z^3 + z^2 + 5z + 6$$

1. By using the formula

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$$

to show that $f'(z) = 6z^2 + 2z$

- Let $z = x + iy$, determine $u(x, y)$ and $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$
- Show that $f(z)$ is analytic on the whole complex plane.
- Verify your answer in (1).

Hint: Cauchy-Riemann equation: $\frac{\partial u(x, y)}{\partial x} = \frac{\partial v(x, y)}{\partial y}$ and $\frac{\partial v(x, y)}{\partial x} = -\frac{\partial u(x, y)}{\partial y}$

4.) Calculate e^{At} for $A = \begin{pmatrix} -2 & 4 \\ -1 & 3 \end{pmatrix}$ (15%)

5.) Find the frequency spectrum of the periodic square wave $f(x)$ and

$$f(x) = \begin{cases} 0 & -1/2 < x < -1/4 \\ 1 & -1/4 < x < 1/4 \\ 0 & 1/4 < x < 1/2 \end{cases} \quad (15\%)$$

6.) Suppose $\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0$ and $u(x, 0) = u(x, b) = u(a, y) = 0$, $u(0, y) = 1$. Find $u(x, y)$ within $0 \leq x \leq a$ and $0 \leq y \leq b$ (20%)