1. Consider the following differential equation (20%)

\[ y'(x) - 3y(x) = 3x + 2 \]

(1) Solve the differential equation. (10%)

(2) Very your answer in (1), by series method at \( x = 0 \). (10%)

2. Solve the following integral equation (15%)

\[ y(t) = 2\cos 2t + e^t \int_0^t y(\lambda) e^{-\lambda} d\lambda \]

3. Use the Laplace transform to solve \( x(t) \) and \( y(t) \) for non-homogeneous linear differential equation system with their initial conditions given by

\[ \begin{cases}
\frac{dx(t)}{dt} - 2x(t) - 3y(t) = 2e^{2t} \\
x(0) = 0, y(0) = 0
\end{cases} \quad \begin{cases}
-x(t) + \frac{dy(t)}{dt} - 4y(t) = 3e^{2t} \\
x(0) = 0, y(0) = 0
\end{cases} \quad (15\%)

4. Using Fourier integral theorem of \( f(x) = e^{-x} \) to show that

\[ \int_0^\infty \frac{\cos \alpha x}{1 + \alpha^2} \, d\alpha = \frac{\pi e^{-x}}{2} \quad (15\%) \]

5. A infinitely long, thin, conducting circular tube of radius \( b \) is split in two halves. The upper half is kept at a potential \( V = V_0 \), and the lower half at \( V = -V_0 \). Determine the potential distribution inside the tube? (20%)  

(Hint: The potential \( V \) satisfy Laplace’s equation \( \nabla^2 V(r, \phi) = 0 \), where \( r, \phi \) are cylindrical coordinates)

6. \[ \int_0^\infty \frac{x^3 \, dx}{(1 + x)^5} = ? \quad (15\%) \]