

科目	工程數學	適用系所	資通訊與電控產業碩士專班	時間	一〇〇分鐘
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※請務必在答案卷作答區內作答。

共 1 頁第 1 頁

1. Consider the following differential equation (20%)

$$y'(x) - 3y(x) = 3x + 2$$

- (1) Solve the differential equation. (10%)
 (2) Verify your answer in (1), by series method at $x = 0$. (10%)

2. Solve the following integral equation (15%)

$$y(t) = 2 \cos 2t + e' \int_0^t y(\lambda) e^{-\lambda} d\lambda$$

3. Use the Laplace transform to solve $x(t)$ and $y(t)$ for non-homogeneous linear differential equation system with their initial conditions given by

$$\begin{cases} \frac{dx(t)}{dt} - 2x(t) - 3y(t) = 2e^{2t} \\ -x(t) + \frac{dy(t)}{dt} - 4y(t) = 3e^{2t} \end{cases}, \quad x(0) = 0, y(0) = 0 \quad (15\%)$$

4. Using Fourier integral theorem of $f(x) = e^{-x}$ to show that $\int_0^{\infty} \frac{\cos \alpha x}{1 + \alpha^2} d\alpha = \frac{\pi}{2} e^{-x}$ (15%)

5. A infinitely long, thin, conducting circular tube of radius b is split in two halves. The upper half is kept at a potential $V = V_0$, and the lower half at $V = -V_0$. Determine the potential distribution inside the tube? (20%)

(Hint: The potential V satisfy Laplace's equation $\nabla^2 V(r, \phi) = 0$, where r, ϕ are cylindrical coordinates)

6. $\int_0^{\infty} \frac{x^3 dx}{(1+x)^5} = ?$ (15%)